B Online Appendix (not for publication)

This Online Appendix provides some additional results referenced in the paper.

B.1 Recovering the equilibrium variables from the Universal Gravity conditions

In this subsection, we show how the universal gravity conditions C.1-C.5 can be combined to derive equations (6) and (7), which can be used to solve for equilibrium prices and price indices up to scale. We then show how information of these prices and price indices up-to-scale can be used to solve for the level of real output prices $\{p_i/P_i\}_{i\in S}$ and, combined with the numeraire in C.6, to determine the equilibrium level of income $\{Y_i\}_{i\in S}$, expenditure $\{E_i\}_{i\in S}$, and trade flows $\{X_{ij}\}_{i,j\in S}$. Finally, we show how all other endogenous variables can be recovered up-to-scale if the equilibrium prices and price indices are known up to scale.

B.1.1 From Universal Gravity C.1-C.5 to Equations (6) and (7)

We first show Universal Gravity C.1-C.5 imply equations (6) and (7).

Combing C.1 and C.2 (in particular the gravity equation (10)):

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j, \tag{31}$$

where recall from C.2 that the price index can be written as:

$$P_i^{-\phi} \equiv \sum_{j\in S} \tau_{ji}^{-\phi} p_j^{-\phi} \tag{32}$$

Combining equation (31) with C.(4) and C.(5) yields:

$$p_i Q_i = \sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j Q_j \tag{33}$$

Finally, we substitute C.3 into equation (33) to yield:

$$p_i \left(C_i \left(\frac{p_i}{P_i} \right)^{\psi} \right) = \sum_{j\in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j \left(C_j \left(\frac{p_j}{P_j} \right)^{\psi} \right) \tag{34}$$

Note that equations (32) and (34) are equivalent to equations (6) and (7). Hence, C.1-C. 5 imply equations (6) and (7), as claimed. There are two things to note about equilibrium equations (32) and (34): first, they depend only on output prices $\{p_i\}$, the price indices $\{P_i\}$, and exogenous model fundamentals (in particular, they do not depend on the endogenous scalar κ); second, they are homogeneous of degree zero with respect to $\{p_i, P_i\}$, so the scale of prices (and price indices) are undetermined.

B.1.2 From Equations (6) and (7) to endogenous variables

We now show that given a solution to equations (6) and (7), we can construct all endogenous variables in the models. We divide the derivations into endogenous variables determined up to scale and endogenous variables for which the scale is known (given the choice of numeraire in C.6.

Suppose that we have a set of prices $\{p_i\}_{i\in S}$ and price indices $\{P_i\}_{i\in S}$ that solve equations (6) and (7). Note that because equations (6) and (7) are homogeneous of degree zero with respect to $\{p_i, P_i\}_{i\in S}$, for any scalar α, the normalized prices $\tilde{p}_i \equiv \frac{1}{\alpha}p_i$ and price indices $\tilde{P}_i \equiv \frac{1}{\alpha}P_i$ continue to satisfy equations (6) and (7).

We first solve for the real output price. Note that for any choice of α, the real output price $\{p_i/P_i\}_{i\in S}$ remains unchanged, so its level is unaffected by the unknown scalar.

We now solve for quantities. From equation (11), the quantity in location i does not depend on α, but it does depend on the unknown scalar κ as follows:

$$Q_i = \kappa C_i \left(\frac{p_i}{P_i}\right)^{\psi}.$$

Hence, equilibrium quantities are only determine up-to-scale.

We now solve for income and expenditure. From C.4 and C.5 we have:

$$E_i = Y_i = p_i Q_i.$$

Applying the numeraire in C.6 then yields:

$$\begin{aligned}
\sum_{i\in S} Y_i = 1 &\iff \\
\sum_{i\in S} p_i Q_i = 1 &\iff \\
\kappa\alpha \sum_{i\in S} \tilde{p}_i C_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi} = 1 &\iff \\
\kappa\alpha &= \left(\sum_{i\in S} \tilde{p}_i C_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi}\right)^{-1},
\end{aligned}$$

which, as claimed, pins down the product of the unknown quantity scalar and unknown price scalar. Given $\kappa\alpha$, we can now determine the level of income and expenditure as follows:

$$\begin{aligned}
E_i = Y_i = p_i Q_i &\iff \\
E_i = Y_i &= \frac{\tilde{p}_i C_i \left(\frac{\tilde{p}_i}{\tilde{P}_i}\right)^{\psi}}{\left(\sum_{j\in S} \tilde{p}_j C_j \left(\frac{\tilde{p}_j}{\tilde{P}_j}\right)^{\psi}\right)},
\end{aligned}$$

as claimed.

We now determine the level of trade flows using equation (31):

$$\begin{aligned}
X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j &\iff \\
X_{ij} &= \frac{\tau_{ij}^{-\phi}\tilde{p}_i^{-\phi}}{\sum_{k\in S} \tau_{kj}^{-\phi}\tilde{p}_k^{-\phi}} \left(\frac{\tilde{p}_j C_j \left(\frac{\tilde{p}_j}{\tilde{P}_j}\right)^{\psi}}{\left(\sum_{k\in S} \tilde{p}_k C_k \left(\frac{\tilde{p}_k}{\tilde{P}_k}\right)^{\psi}\right)}\right).
\end{aligned}$$

Other than real output prices $\{p_i/P_i\}_{i\in S}$, income $\{Y_i\}_{i\in S}$, expenditure $\{E_i\}_{i\in S}$, and trade flows

$\{X_{ij}\}_{i,j \in S}$, all other endogenous variables are determined only up-to-scale, as they depend either on the price scalar α (i.e. output prices $\tilde{p}_i$, price indices $\tilde{P}_i$, bilateral prices $p_{ij} = \tau_{ij}\tilde{p}_i$, and the quantity traded $Q_{ij} = X_{ij}/\tau_{ij}p_i$) or the quantity scalar κ (i.e. quantities Q_i).

B.2 Proof of Theorem 1 part (ii)

We first provide a general mathematical formulation to incorporate non-interior solutions. Let the equilibrium be a duple $(p_i, Q_i) \in \overline{\mathbb{R}}_+^N \times \mathbb{R}_+^N$ such that for all $i \in S$,

$$Q_i = \sum_j \frac{\tau_{ij}^{-\phi-1} p_i^{-\phi-1}}{\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j \tag{35}$$

$$(p_i, Q_i) \in F_i(p, Q) \tag{36}$$

where F is a supply condition, which might be a correspondence. (The fact that F might be correspondence allows us to extend the framework to allow for non-interior solutions). In particular, we define F as follows: We say $(p_i, Q_i) \in F_i(p, Q)$ if and only if

$$sign(\psi) \left[Q_i - \kappa \left(\frac{p_i}{P_i(p)} \right)^{\psi} \right] \geq 0 \tag{37}$$

$$Q_i = \kappa \left(\frac{p_i}{P_i(p)} \right)^{\psi} \quad \text{if } Q_i > 0, \tag{38}$$

and where $\left(\frac{0}{0}\right)$ is defined as 0. That is, if $Q_i = 0$, then we replace C.3 with an inequality. For example, in an economic geography model, inequality constraint (37) corresponds to welfare equalization. If there are people living in location i, then Q_i is given by equality (38). If not, then the welfare living in location i should be lower than one obtained in other places, which is represented as the inequality (37).

As we mentioned in Section 3, we restrict our attention to non-trivial equilibria where there is positive production in at least one location. To show that all (non-trivial) equilibria are interior, it then suffices to show that if some locations produces nothing, then all other locations must also produce nothing.

Suppose that $Q_l = 0$ for some $l \in S$. Then from equation (35) for l:

$$0 = \sum_j \underbrace{\frac{\tau_{lj}^{-\phi} p_l^{-\phi-1}}{\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j}_{\geq 0}, \tag{39}$$

which in turn implies that for all $j \in S$,

$$\frac{\tau_{lj}^{-\phi} p_l^{-\phi-1}}{g_j} p_j Q_j = 0, \tag{40}$$

where $g_j = \sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}$.

Note that there are two reasons why equation (40) is zero for all j; either (1) ; or (2) for all $j \in S$, $\tau_{lj}^{-\phi} \frac{p_j Q_j}{g_j} = 0$. We will prove a contradiction in both cases.

First assume that (1) $p_l^{-\phi-1} = 0$, which if $\phi > -1$ implies that $p_l = \infty$. While $(p_l, Q_l) = (\infty, 0)$ satisfies equation (39), it does not satisfy equation (36). To see this, note:

$$0 = Q_i < \kappa \left(\frac{p_i}{g_i^{-\frac{1}{\phi}}} \right)^{\psi} = \infty,$$

which contradicts with equation (37) since $\psi \geq 0$. Therefore p_l needs to be finite, $p_l < \infty$.

Now assume that (2) for all $j \in S$, $\tau_{lj}^{-\phi} \frac{p_j Q_j}{g_j} = 0$. Since the price for country l is finite, equation (40) is reduced to

$$\tau_{lj}^{-\phi} \frac{p_j Q_j}{g_j} = 0$$

for all $j \in S$. An equivalent expression is that for all countries connected with l, $j \in S_l = \{k \in S; \tau_{lk} < \infty\}$,

$$p_j Q_j = 0 \quad \text{or} \quad g_j = \infty. \tag{41}$$

Fix any country $j \in S_l$. Suppose that $p_j, Q_j > 0$ Then equation (41), $g_j = \infty$. Then for all $(p_j, Q_j) \in \overline{\mathbb{R}}_+ \times \mathbb{R}$ if $\psi \geq 0$ we have

$$\infty = \kappa \left(\frac{p_j}{g_j^{-\frac{1}{\phi}}} \right)^{\psi} \leq Q_j = 0,$$

which is a contradiction. Therefore in order to satisfy equation (41), p_j or Q_j needs to be zero. Suppose that $p_j = 0$. Then we have

$$0 = \kappa \left(\frac{p_j}{g_j^{-\frac{1}{\phi}}} \right)^{\psi} \leq Q_j.$$

If $Q_j > 0$, then C. (3). Therefore, $Q_j = 0$. Therefore Q_j needs to be zero for all $j \in S_l$.

So far, we have shown that if $Q_l = 0$ then the connected countries $j \in S_l$ produce nothing, $Q_j = 0$. Because of strong connectedness, any country n is connected with l through third countries. Therefore, by repeating the argument along with the chain, we have $Q_n = 0$ for all $n \in S$.

As a result, if $\phi \geq -1$, and $\psi \geq 0$ then all equilibria are interior, as claimed.

B.3 Quasi-symmetric trade frictions

In this subsection, we show that when trade frictions are quasi-symmetric, then balanced trade implies that the origin and destination fixed effects of the gravity trade flow expression are equal up to scale.

We first formally define "quasi-symmetry." We say that the set of trade frictions $\{\tau_{ij}\}_{i,j \in S}$ are *quasi-symmetric* if there exists a set of origin scalars $\left\{\tau_i^A\right\}_{i \in S} \in \mathbb{R}_{++}^N$, destination scalars $\left\{\tau_i^B\right\}_{i \in S} \in \mathbb{R}_{++}^N$, and a symmetric matrix $\{\tilde{\tau}_{ij}\}_{i,j \in S}$ where $\tilde{\tau}_{ij} = \tilde{\tau}_{ji}$ for all $i, j \in S$ such that we can write:

$$\tau_{ij} = \tau_i^A \tau_i^B \tilde{\tau}_{ij} \ \forall i, j \in S.$$

Loosely speaking, quasi-symmetric trade frictions are those that are reducible to a symmetric component and exporter- and importer-specific components. While restrictive, it is important to note that the vast majority of papers which estimate gravity equations assume that trade frictions are quasi-symmetric; for example Eaton and Kortum (2002) and Waugh (2010) assume that trade frictions are composed by a symmetric component that depends on bilateral distance and on a destination or origin fixed effect.

Combining the universal gravity conditions C. 1 and C. 2 allows us to write the value of bilateral trade flows from i to j as:

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j,$$

which we now re-write as:

$$X_{ij} = \kappa \tau_{ij}^{-\phi} \gamma_i \delta_j, \tag{42}$$

where we call $\gamma_i \equiv p_i^{-\phi}$ the *origin fixed effect* and $\delta_i \equiv P_i^{\phi} E_i = C_i P_i^{\phi-\psi} p_i^{1+\psi}$ the *destination fixed effect*.

Proposition 1. *If trade frictions are quasi-symmetric, then in any model within the universal gravity framework, the product of the equilibrium origin fixed effect and the origin scalar will be equal to the product of the equilibrium destination fixed effects and the destination fixed effect up to scale, i.e.: for some scalar* $\lambda \geqslant 0$,

$$\left(\tau_i^A\right)^{-\phi} \gamma_i = \lambda \left(\tau_i^B\right)^{-\phi} \delta_i \ \forall i \in S.$$

Proof. We first note that market clearing condition C.4 and balanced trade condition C.5 together imply that: $\sum_{j\in S} X_{ij} = \sum_{j \in S} X_{ji}$ $\forall i \in S$. Combining this with the gravity expression (42) and quasi-symmetry implies:

$$\sum_j \underbrace{\kappa \tau_{ij}^{-\phi} \gamma_i \delta_j}_{=X_{ij}} = \sum_j \underbrace{\kappa \tau_{ji}^{-\phi} \gamma_j \delta_i}_{X_{ji}} \iff$$

$$\frac{\left(\tau_i^A\right)^{-\phi} \gamma_i}{\left(\tau_i^B\right)^{-\phi} \delta_i} = \frac{\sum_{j\in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_j^A\right)^{-\phi} \gamma_j}{\sum_{j\in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_j^B\right)^{-\phi} \delta_j} = \sum_{j\in S} \frac{\tilde{\tau}_{ij}^{-\phi} \left(\tau_j^B\right)^{-\phi} \delta_j}{\sum_{k\in S} \tilde{\tau}_{ik}^{-\phi} \left(\tau_k^B\right)^{-\phi} \delta_k} \times \frac{\left(\tau_j^A\right)^{-\phi} \gamma_j}{\left(\tau_j^B\right)^{-\phi} \delta_j}.$$

It is easy to show that $\frac{\left(\tau_i^A\right)^{-\phi} \gamma_i}{\left(\tau_i^B\right)^{-\phi} \delta_i} = 1$ is a solution to this problem for any kernel. From the Perron-Frobenius theorem, the solution is unique up to scale. Therefore we have:

$$\left(\tau_i^A\right)^{-\phi} = \lambda \left(\tau_i^B\right)^{-\phi} \delta_i \ \forall i \in S, \tag{43}$$

as required. □

Proposition 1 has a number of important implications. First, Proposition 1 allows one to simplify the equilibrium system of equations 6 and 7 into a single non-linear equation when $\phi \neq \psi$:

$$\left(p_i^{\frac{1+\psi+\phi}{\psi-\phi}}\right)^{-\phi} = (\lambda)^{\frac{\phi}{\psi-\phi}} (C_i)^{\frac{\phi}{\psi-\phi}} \sum_{j\in S} \tilde{\tau}_{ij}^{-\phi} \left(\tau_i^A\right)^{\frac{\phi^2}{\psi-\phi}} \left(\tau_i^B\right)^{-\frac{\phi\psi}{\psi-\phi}} \left(\tau_j^A\right)^{-\phi} p_j^{-\phi}, \quad i \in S, \tag{44}$$

which simplifies the characterization of the theoretical and empirical properties of the equilibrium.

Notice that λ is an endogenous scalar. Since (44) holds for any location $i \in S$, λ is expressed as

$$\lambda^{\frac{\phi}{\psi-\phi}} = \frac{\sum_i \left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}}{\sum_i \sum_{j\in S} \tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} C_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}.$$

Substituting above expression, we obtain:

$$\frac{\left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}}{\sum_i \left(p_i^{-\phi}\right)^{\frac{1+\psi+\phi}{\psi-\phi}}} = \sum_{j\in S} \frac{\tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} C_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}{\sum_i \sum_{j\in S} \tau_{ij}^{-\phi} \left(\frac{\tau_i^A}{\tau_i^B}\right)^{\frac{\phi^2}{\psi-\phi}} C_i^{\frac{\phi}{\psi-\phi}} p_j^{-\phi}}.$$

Notice that the system is now homogeneous degree 0. Therefore, if $\phi \notin \left\{-\frac{1}{2}, \psi, 0\right\}$, then we can normalize $\lambda = 1$ without loss of generality.

Second, by showing that the origin and destination fixed effects are equal up to scale, Proposition 1 provides offers an analytical characterization of the equilibrium. For example, given the definition of the origin and destination fixed effects, Proposition 1 can equivalently be expressed as:

$$p_i P_i \propto \frac{\tau_i^B}{\tau_i^A} E_i^{-\frac{1}{\phi}}, \tag{45}$$

i.e. there is a log-linear relationship between output prices, the price index and total expenditure in a location.

Third, it is straightforward to show that quasi-symmetry implies that equilibrium trade flows will be bilaterally symmetric, i.e. $X_{ij} = X_{ji}$ for all $i, j \in S$, allowing one to test whether trade frictions are quasi-symmetric directly from observed trade flow data.

Finally, we should note that the results of Proposition 1 have already been used in the literature for particular models, albeit implicitly. The most prominent example is Anderson and Van Wincoop (2003), who use the result to show the bilateral resistance is equal to the price index.[36] To our knowledge, Head and Mayer (2013) are the first to recognize the importance of balanced trade and market clearing in generating the result for the Armington model; however, Proposition 1 shows that the result applies more generally to any model with quasi-symmetrical trade frictions in the universal gravity framework.

B.4 Proofs of the lemmas used in Theorem (1)

There are 4 lemmas which are not proven in the paper. In this section, we discuss them carefully. Before proving these lemmas, we discuss how we use them in the proof. In the proof, we show a fixed point for the "scaled" system, not the actual system. Therefore it needs to be shown that there exists a fixed point for the actual system, which is shown in Lemma 1. Then we argue that the solution we obtain is strictly positive, which is guaranteed by Assumption 1. We emphasize the connectivity assumption is crucial here. These two lemmas are used in **Part i)** Theorem 1.

Part ii) shows that there exists an unique solution. During the proof, we argue that 26 should hold with strict inequality. Again the connectivity allows us to show this result (Lemma 3). After

[36] The result is also used in economic geography byAllen and Arkolakis (2014) to simplify a set on non-linear integral equations into a single integral equation.

establishing this strict inequality, we follow the argument by Allen et al. (2014), which requires that the largest absolute eigenvalues for $|A|$ are less than 1. Since A is a 2-by-2 matrix, we can compute the eigenvalues by hand and show that one of them is exactly 1, and the other is less than 1 if the conditions in **Part ii)** are satisfied.

Lemma 1. *Suppose that z solves (22). Then there exists $\widehat{z}$ solving (21).*

Proof. First it is easy to show[37]

$$\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}} = \sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}. \tag{48}$$

Guess a solution

$$\widehat{z} = \begin{pmatrix} (\widehat{x}_i)_i \\ (\widehat{y}_i)_i \end{pmatrix} = \begin{pmatrix} t^{-1}\,(x_i)_i \\ t^{-1}\,(y_i)_i \end{pmatrix}, \tag{49}$$

where $t = \left(\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{21}} y_j^{a_{22}}\right)^{\frac{1}{1-a_{11}-a_{12}}} = \left(\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}\right)^{\frac{1}{1-a_{21}-a_{22}}}$.[38] Then it is easy to verify that (49) solves (21); in particular, note that

$$\begin{aligned} \widehat{x}_i &= t^{-1}\frac{\sum_{j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}} = t^{1-a_{11}-a_{12}}\frac{\sum_{j\in S} K_{ij}C_i^{-1}C_j\,(\widehat{x}_j)^{a_{11}}\,\widehat{y}_j^{a_{12}}}{\sum_{i,j\in S} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}} \\ &= \sum_{j\in S} K_{ij}C_i^{-1}C_j \widehat{x}_j^{a_{11}} \widehat{y}_j^{a_{12}}. \end{aligned}$$

We can also show that the second equations in (21) are also solved in the same vein:

$$\begin{aligned} \widehat{y}_i &= t\frac{\sum_{j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}}{\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} = t^{1-a_{21}-a_{22}}\frac{\sum_{j\in S} K_{ji} \widehat{x}_j^{a_{21}} \widehat{y}_j^{a_{22}}}{\sum_{i,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}} \\ &= \sum_{j\in S} K_{ji} \widehat{x}_j^{a_{21}} \widehat{y}_j^{a_{22}}. \end{aligned}$$

The above two equations confirm that $\widehat{x}_i$ and $\widehat{y}_i$ is a solution to (21). □

[37] To see this, multiply $C_i x_i^{a_{21}} y_i^{a_{22}} = C_i p_i^{-\phi}$, to the first equations of (22) and sum over i;

$$\sum_i C_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_i\sum_j K_{ij}C_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i,j} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}}}. \tag{46}$$

Also multiply $C_i x_i^{a_{11}} y_i^{a_{12}} = C_i P_i^{\phi-\psi} p_i^{1+\psi}$ to the second equations (22) and sum over i;

$$\sum_{i\in S} C_i p_i^{1+\psi} P_i^{-\psi} = \frac{\sum_{i\in S}\sum_{j\in S} K_{ij}C_j x_i^{a_{21}} y_i^{a_{22}} x_j^{a_{11}} y_j^{a_{12}}}{\sum_{i\in S,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}}. \tag{47}$$

Notice that the LHS is the same as one in (46). Also the numerator of the RHS in (46) is the same as one in (47). Therefore the following double sum terms should be the same:

$$\sum_{i,j} K_{ij}C_i^{-1}C_j x_j^{a_{11}} y_j^{a_{12}} = \sum_{i\in S,j\in S} K_{ji} x_j^{a_{21}} y_j^{a_{22}}.$$

[38] Notice that $a_{11} + a_{12} = a_{21} + a_{22}$.

Lemma 2. *If $\{\tau_{ij}\}_{i,j}$ satisfies Assumption 1, then the fixed point for (22) is strictly positive.*

Proof. We need to consider four different cases for the combinations of a_{11}, a_{12} satisfying different inequalities. We will consider the case $a_{11}, a_{12} > 0$ since the logic in the other cases is the same. We proceed by contradiction. Suppose that there is a solution x to equation (22) such that for some $i \in S$ $x_i = 0$. Consider an arbitrary location $n \neq i$ and consider a connected path, $K_{in}^c \equiv K_{i\pi_1} \times ... \times K_{\pi_m n} > 0$ for some $m(*)$. Then, from the first of equations in (21) notice that

$$x_i = \sum_{j \in S} K_{ij} x_j^{a_{11}} y_j^{a_{12}} \geq \underbrace{K_{i\pi_1}}_{\neq 0} x_{\pi_1}^{a_{11}} y_{\pi_1}^{a_{12}}.$$

Note that $K_{i\pi_1}$ is strictly positive due to $(*)$. Then either x_n or y_n or both are zero if a_{11} and $a_{11} > 0$. If$x_n = 0$ this argument holds for any$n$ so this is a contradiction with the non-zero equilibrium proved above. Else if $y_n = 0$ we can repeat the argument the second of the equations in (21) to establish another contradiction. Notice that if either of $\alpha_{11}, \alpha_{12} = 0$ a contradiction is also easy to establish. □

Lemma 3. *Equation 26 holds with strict inequality.*

To that end, define the set of directly connected countries to each location $i \in S$ as $S_i^c \equiv \{j \in S : K_{ij} > 0\}$. Then notice that equation (24) combined with our equality assumption on equation (26) yields

$$\frac{x_i}{\widehat{x}_i} = \frac{1}{\widehat{x}_i} \sum_{j \in S_i^c} K_{ij} C_i^{-1} C_j \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} (\widehat{x}_j)^{\alpha_{11}} (\widehat{y}_j)^{\alpha_{12}} = \max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

Notice that given that $\hat{x}_i$ is a solution, this implies that the following has to be true for all $j \in S_i^c$

$$\left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} = \max_{j \in S} \left(\frac{x_j}{\widehat{x}_j}\right)^{\alpha_{11}} \quad \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}} = \max_{j \in S} \left(\frac{y_j}{\widehat{y}_j}\right)^{\alpha_{12}}.$$

Now notice that if $\alpha_{11} \neq 0$ then for all $n \in S_i^c$,$x_j/\widehat{x}_j = x_n/\widehat{x}_n$. However, because of C. 1, we assume that there exists an indirectly connected path from any location to any other location, so that repeating this argument for all j and using the indirect connectivity we can prove that $x_j/\widehat{x}_j = x_n/\widehat{x}_n$ for all $j, n \in S$ i.e. the solutions are the same up-to-scale, a contradiction.

Lemma 4. *If $\phi, \psi \geqslant 0$ or $\phi, \psi \leqslant -1$, the eigenvalue for $|A|$ is*

$$\lambda = \frac{\phi - \psi}{1 + \phi + \psi}, 1,$$

and

$$\left|\frac{\phi - \psi}{1 + \phi + \psi}\right| < 1.$$

Proof. Notice that

$$|A| = \begin{pmatrix} \left|\frac{1+\psi}{1+\psi+\phi}\right| & \left|\frac{1+\phi}{1+\psi+\phi}\right| \\ \left|\frac{\phi}{1+\psi+\phi}\right| & \left|\frac{\psi}{1+\psi+\phi}\right| \end{pmatrix} = \begin{pmatrix} \frac{1+\psi}{1+\psi+\phi} & \frac{1+\phi}{1+\psi+\phi} \\ \frac{\phi}{1+\psi+\phi} & \frac{\psi}{1+\psi+\phi} \end{pmatrix}.$$

Then we can solve the following characteristic functions

$$\lambda^2 - \left(\frac{1+\psi}{1+\psi+\phi} + \frac{\psi}{1+\psi+\phi}\right)\lambda + \frac{1+\psi}{1+\psi+\phi}\frac{\psi}{1+\psi+\phi} - \frac{1+\phi}{1+\psi+\phi}\frac{\phi}{1+\psi+\phi} = 0.$$

Then

$$\lambda = \frac{\phi - \psi}{1 + \phi + \psi}, 1.$$

We need to show that $\left|\frac{\phi-\psi}{1+\phi+\psi}\right| < 1$. To show it, it suffices to show

$$g = |1 + \phi + \psi| - |\phi - \psi| > 0$$

Suppose that $\phi, \psi \geq 0$. Then g is strictly positive as follows:

$$\begin{aligned} g &= 1 + \phi + \psi - |\phi - \psi| \\ &\geq 1 + \phi + \psi - (|\phi| + |\psi|) = 1 > 0. \end{aligned}$$

Suppose that $\phi, \psi \leq -1$. Then g is given by

$$g = -1 - \phi - \psi - |\phi - \psi|.$$

If $\phi \leq \psi$, then

$$\begin{aligned} g &= -1 - \phi - \psi + \phi - \psi \\ &= -1 - 2\psi \geq 1. \end{aligned}$$

If $\phi \geq \psi$, then

$$\begin{aligned} g &= -1 - \phi - \psi - \phi + \psi \\ &= -1 - 2\psi \geq 1, \end{aligned}$$

which completes the proof. □

B.5 Lemmas and Proposition used in Theorem 2 (iii)[39]

In this section, we prove the lemma and proposition used in Theorem 2 (iii).

Lemma 5. *If $\phi, \psi \geq 0$ or $\phi, \psi \leq -1$, then A has strictly positive diagonal elements and is diagonal dominant in its rows; namely, for all $i \in S$*

$$A_{ii} > 0, \tag{50}$$

$$|A_{ii}| > \sum_{j \in S - i} |A_{ij}|. \tag{51}$$

Proof. Recall that A matrix is

$$A = \mathbf{Y} + \frac{\phi - \psi}{1 + \psi + \phi}\mathbf{X},$$

and from Lemma 4,

$$\left|\frac{\phi - \psi}{1 + \phi + \psi}\right| < 1.$$

[39] A similar argument is found in Johnson and Smith (2011).

Then the diagonal elements for A are positive; for all $i \in S$,

$$\begin{aligned} A_{ii} &= Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi} X_{ii} \\ &= Y_{ii} - \left| \frac{\phi - \psi}{1 + \psi + \phi} \right| X_{ii} \\ &> Y_{ii} - X_{ii} \geq 0. \end{aligned}$$

Also, for all $i \in S$,

$$\begin{aligned} & |A_{ii}| - \sum_{l \in S - i} |A_{il}| \\ =& \left| \underbrace{Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi} X_{ii}}_{>0} \right| - \left| \frac{\phi - \psi}{1 + \psi + \phi} \right| \sum_{l \in S - i} X_{il} \\ =& Y_{ii} + \frac{\phi - \psi}{1 + \psi + \phi} X_{ii} - \left| \frac{\phi - \psi}{1 + \psi + \phi} \right| (Y_i - X_{ii}) \\ =& \left(\underbrace{1 - \left| \frac{\phi - \psi}{1 + \psi + \phi} \right|}_{>0} \right) Y_{ii} + \left[\underbrace{\frac{\phi - \psi}{1 + \psi + \phi} + \left| \frac{\phi - \psi}{1 + \psi + \phi} \right|}_{\geq 0} \right] X_{ii} > 0, \end{aligned}$$

which is equation (51). □

The next proposition plays a crucial role in the proof for Theorem 2 (iii).

Proposition 2. *If A has strictly positive diagonal elements and is dominant of its rows, then for all $i \neq j$,*

$$A_{ii}^{-1} > A_{ji}^{-1} > 0.$$

Proof. The co-factor expansion of A^{-1} is[40]

$$\begin{aligned} A_{ii}^{-1} - A_{ji}^{-1} &= \frac{det\left(A\left[S - i\right]\right) - (-1)^{i+j} det\left(A\left[S - i, S - j\right]\right)}{det\left(A\right)} \\ &= \frac{det\left(T\right)}{det\left(A\right)}, \end{aligned}$$

where T is defined as follows:

$$\widetilde{T} = A + \left(\underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{N \times (j-1)}, \underbrace{A_i}_{N \times 1}, I \underbrace{\mathbf{0}, \cdots, \mathbf{0}}_{N \times (N-j)} \right).$$

[40]Remember

$$A_{ij}^{-1} = (-1)^{i+j} \frac{det\left(A\left[N - j, N - i\right]\right)}{det\left(A\right)}.$$

T is a principal component of $\widetilde{T}$:

$$T = \widetilde{T}\left[S-i, S-i\right].$$

If a matrix C has positive diagonal elements, and is diagonally dominant of its rows, then $det\left(C\right) > 0$. [41] Then if T has such properties, then

$$\frac{det\left(T\right)}{det\left(A\right)} > 0$$

since A is assumed to have these properties. Thus it suffices to show that T has positive diagonal elements and is dominant of its rows.

By construction of T, it suffices to show

$$A_{kk} > 0 \quad k \in S-i-j \tag{52}$$

$$A_{kk} + A_{ki} > 0 \quad k = j \tag{53}$$

$$|A_{kk}| > \sum_{l \in S-i-k} |A_{kl} + 1_{l=j} A_{ki}| \quad k \in S-i-j \tag{54}$$

$$|A_{kk} + A_{ki}| > \sum_{l \in S-i-k} |A_{kl}| \quad k = j. \tag{55}$$

First we show equation (52) and equation (53). since A has a strictly positive diagonal, for all $k \in S$,

$$A_{kk} > 0,$$

which is equation (52) . Also since A is diagonal dominant,

$$A_{jj} + A_{ji} > \sum_{l \neq j} |A_{jl}| + A_{ji} \geq |A_{ji}| + A_{ji} \geq 0,$$

which is equation (53).

Second, we show equation (54) and equation (55). Fix $k \in N-i-j$. Since A is diagonally dominant,

$$\begin{aligned}
|A_{kk}| &> \sum_{l \in S-k} |A_{kl}| \\
&= \sum_{l \in S-k-i-j} |A_{kl}| + |A_{ki}| + |A_{kj}| \\
&\geq \sum_{l \in S-i-k-j} |A_{kl}| + |A_{ki} + A_{kj}| \quad (\because \text{triangle inequality}) \\
&= \sum_{l \in S-i-k} |A_{kl} + 1_{l=j} A_{ki}|,
\end{aligned}$$

which is equation (54). Fix $k = j$. Since A has positive diagonal elements, and is diagonally

[41] See also Theorem 3 of Evmorfopoulos (2012).

dominant,

$$\begin{aligned}
|A_{kk} + A_{ki}| &\geqslant ||A_{kk}| - |A_{ki}|| \\
&= |A_{kk}| - |A_{ki}| \quad \left(\because |A_{kk}| \geq \sum_{l \in S-k} |A_{kl}| \geqslant |A_{ki}|\right) \\
&= \sum_{l \in S-k-i} |A_{kl}| + |A_{ki}| - |A_{ki}| \\
&= \sum_{l \in S-k-i} |A_{kl}|,
\end{aligned}$$

which is equation (55). □

B.6 Existence and Uniqueness using Gross Substitutes Methodology (a la Alvarez and Lucas (2007))

In this subsection, we prove the existence and uniqueness of an equilibrium in our universal gravity framework using the gross substitutes methodology employed by Alvarez and Lucas (2007). As we show below, the sufficient conditions here are stronger than we provide in Theorem 1 above.

Proposition 3. *Consider any model within the universal gravity framework. If $\phi > \psi > 0$ and $\tau_{ij} \in (0, \infty)$ for all $i, j \in S$, then the excess demand system of the model satisfies gross substitutes and, as a result, the equilibrium exists and is unique.*

Proof. Recall the equilibrium conditions of the universal gravity framework from equations (6) and

$$p_i C_i \left(\frac{p_i}{P_i}\right)^{\psi} = \sum_{j \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} p_j C_j \left(\frac{p_j}{P_j}\right)^{\psi} \forall i \in S \tag{56}$$

$$P_i^{-\phi} = \sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \; \forall i \in S \tag{57}$$

Substituting equation (57) into (56) yields a single equilibrium system of equations that depends only on the output prices in every location:

$$p_i^{1+\phi+\psi} \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i = \sum_{j \in S} \tau_{ij}^{-\phi} C_j p_j^{1+\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} \quad \forall i \in S$$

We define the corresponding excess demand function as:

$$Z_i(\mathbf{p}) = \frac{1}{p_i} \left(\frac{1}{\sum_{k \in S} C_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}}\right) \times \left[\sum_{j \in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi} \left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi} \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i\right], \tag{58}$$

whereP_i is defined by equation (57).This system written as such needs to satisfy 6 properties to be an excess demand system and the gross substitute property to establish existence and uniqueness. The six conditions are:

1.$Z(\mathbf{p})$ is continuous for$\mathbf{p} \in \Delta\left(R_{+}^{N}\right)$

2.$Z(\mathbf{p})$ is homogeneous of degree zero.

3.$Z(\mathbf{p}) \cdot \mathbf{p} = 0$ (Walras' Law).

4. There exists a$k > 0$ such that$Z_j(\mathbf{p}) > -k$ for all$j$.

5. If there exists a sequence $p^m \to p^0$, where $p^0 \neq 0$ and $p_i^0 = 0$ for some i, then it must be that:

$$max_j \left\{Z_j\left(p^m\right)\right\} \to \infty \tag{59}$$

and the gross-substitute property:

6. Gross substitutes property:$\frac{\partial Z(p_j)}{\partial p_k} > 0$ for all$j \neq k$.

We verify each of these properties in turn. Property 1 is trivial given equation (58) for excess demand. To see property 2, consider multiplying output prices by a scalar $\beta > 0$, which immediately yields $Z_i(\beta\mathbf{p}) = Z_i(\mathbf{p})$.as required. Property 3 can be seen as follows:

$$\begin{aligned}
Z(\mathbf{p}) \cdot \mathbf{p} = &= \sum_{i \in S} Z_i(\mathbf{p})\, p_i \iff \\
&= \left(\frac{1}{\sum_{k \in S} C_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}}\right) \times \\
&= \sum_{i \in S}\left(\sum_{j \in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi}\left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi}\left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i\right) \iff \\
&= 0,
\end{aligned}$$

as required. Property 4 can be seen as follows:

$$\begin{aligned}
Z_i(\mathbf{p}) &= \frac{1}{p_i} \frac{\sum_{j \in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi}\left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k \in S} C_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}} - Q_i \implies \\
Z_i(\mathbf{p}) &> -Q_i > \bar{Q}
\end{aligned}$$

since $\frac{1}{p_i}\left(\frac{1}{\sum_{k \in S} C_k \left(\sum_{l \in S} \tau_{lk}^{-\phi} (\beta p_l)^{-\phi}\right)^{\frac{\psi}{\phi}} (\beta p_k)^{\psi}}\right) \sum_{j \in S} \tau_{ij}^{-\phi} C_j p_i^{-\phi} p_j^{1+\psi}\left(\sum_{k \in S} \tau_{kj}^{-\phi} p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} > 0$ for all $\mathbf{p} \gg 0$ and $Q_i \leq \bar{Q}$ from C. 3. Property 5 can be seen as follows: consider any $\mathbf{p} \in \Delta\left(R_{+}^{N}\right)$ such that there exists an$l \in S$ where$p_l = 0$ and an$l' \in S$ where$p_{l'} > 0$. Consider any sequence of output prices such that $\mathbf{p}^n \to \mathbf{p}$ as$n \to \infty$. Then we need to show that:

$$\max_{i \in S} Z_i(\mathbf{p}) \to \infty.$$

To see this note that:

$$\max_{i\in S} Z_i\left(\mathbf{p}^n\right) = \max_{i\in S} \frac{\frac{1}{p_i}\sum_{j\in S}\left(\tau_{ij}p_i\right)^{-\phi} C_j p_j^{1+\psi}\left(\sum_{k\in S}\left(\tau_{kj}p_k\right)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} - Q_i \implies$$

$$\max_{i\in S} Z_i\left(\mathbf{p}^n\right) > \max_{i,j\in S} \frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} - \bar{Q}.$$

Hence, if it is the case that $\max_{i,j\in S} \frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} \to \infty$, then because $\max_{i\in S} Z_i\left(\mathbf{p}^n\right)$ is bounded below it, it must be that $\max_{i\in S} Z_i\left(\mathbf{p}^n\right) \to \infty$ as well. Note that:

$$\max_{i,j\in S} \frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}p_l^{-\phi}\right)^{\frac{\psi}{\phi}} p_k^{\psi}} > \max_{i,j\in S} \frac{p_j}{p_i}\tau_{ij}^{-\phi}\frac{C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi}{\phi}} \left(p^{\max}\right)^{\psi}} \implies$$

$$> C_{ij}\min_{l\in S} p_l^{-(\phi-\psi)},$$

where $p^{\min} \equiv \min_{l\in S} p_l$,$p^{\max} \equiv \max_{l\in S}$, and $C_{ij} \equiv \tau_{ij}^{-\phi}\frac{C_j\left(\sum_{k\in S}\tau_{kj}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi-\phi}{\phi}}}{\sum_{k\in S} C_k\left(\sum_{l\in S}\tau_{lk}^{-\phi}\left(p^{\min}\right)^{-\phi}\right)^{\frac{\psi}{\phi}}\left(p^{\max}\right)^{\psi}}$. Since $\phi > \psi > 0$ and there exists an$l \in S$ such that $p_l^n \to \infty$ as $n \to \infty$, then we have $\max_{i\in S} Z_i\left(\mathbf{p}^n\right) \to \infty$ as well.

Finally, we verify gross-substitutes. Without loss of generality, we differentiate only the bracketed term (as the term outside the bracket will be multiplied by zero since the bracket term is equal to zero in the equilibrium). We have:

$$\begin{aligned}
\frac{\partial Z_i\left(\mathbf{p}\right)}{\partial p_j} =& \frac{\partial}{\partial p_j}\left[\sum_{j\in S}\tau_{ij}^{-\phi}C_j p_i^{-\phi}p_j^{1+\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} - p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}} C_i\right] \iff \\
=& \left(1+\psi\right)\tau_{ij}^{-\phi}C_j p_i^{-\phi}p_j^{\psi}\left(\sum_{k\in S}\tau_{kj}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}} + \\
& \left(\phi-\psi\right)p_j^{-\phi-1}\sum_{l\in S}\tau_{il}^{-\phi}C_l p_i^{-\phi}p_l^{\psi}\left(\sum_{k\in S}\tau_{kl}^{-\phi}p_k^{-\phi}\right)^{\frac{\psi-\phi}{\phi}-1} + \psi p_j^{-\phi-1}p_i^{1+\psi}\left(\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}\right)^{\frac{\psi}{\phi}-1} > 0
\end{aligned}$$

because $\phi > \psi > 0$ and prices, trade frictions, and supply shifters C_l are strictly positive. Because properties 1-6 hold, by Propositions 17.B.2, 17.C.1 and 17.F.3 of Mas-Colell et al. (1995), the equilibrium exists and unique. □

Note that in the case where $\psi > \phi > 0$ – which is the ordering we find when we estimate the gravity constants in Section 5 – Theorem 1 still proves existence and uniqueness of the equilibrium. The following example shows that gross substitutes may not be satisfied in this case.

Example 1. (Gross substitution) Consider the three location economy. Take p_3 as the numeraire The gross substitute is violated if there exists $\overline{p}_1$ such that $Z_1(\overline{p}_1, p_2, 1)$ is not monotonic w.r.t. p_2. Consider the following parameter values:

$$\begin{aligned}(\phi, \psi) &= (2, 5) \\ \tau_{ij} &= 1 \quad \text{for } i, j \in \{1, 2, 3\} \\ C_i &= (.9, .6, .1)^{\mathbf{T}}.\end{aligned}$$

Figure 12 shows that with these parameter values, $Z_1(\overline{p}_1, p_2, 1)$ is not monotonic w.r.t. p_2 when $\overline{p}_1 = .5$.

B.7 Examples of multiplicity in two location world

In this subsection, we derive the equilibrium conditions of a two location world and provide examples for different combinations of the gravity constants (i.e. the demand elasticity ϕ and supply elasticity ψ).

We first derive equations for the demand and supply of the representative good in each location as a function of parameters and prices in all other locations. Combining C. 2 (aggregate demand) and C. 3 (market clearing) yields the following aggregate demand equation:

$$Q_i^d = p_i^{-(1+\phi)} \times \left(\sum_{j \in S} \frac{\tau_{ij}^{-\phi}}{\sum_k \tau_{kj}^{-\phi} p_k^{-\phi}} p_j Q_j^d \right), \tag{60}$$

where we denote the quantity of the representative good demanded in location i as Q_i^d. Similarly, C. 3 (aggregate supply) yields the following aggregate supply equation:

$$Q_i^s = \kappa C_i \left(\frac{p_i}{\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi}} \right)^{\psi}, \tag{61}$$

where we denote the quantity of the representative good supplied in location i as Q_i^d.

Now consider the two-location case (i.e. $S \equiv \{1, 2\}$) where $\tau_{12} = \tau_{21} = \tau \geq 1$ and $C_1 = C_2 = 1$. Dividing Q_1^d by Q_2^d using equation (60) delivers the following relative demand equation:

$$\frac{Q_1^d}{Q_2^d} = \left(\frac{p_1}{p_2}\right)^{-(1+\phi)} \times \frac{\left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi}+1}{\left(\frac{p_1}{p_2}\right)^{-\phi}+\tau^{-\phi}}\right) \times \frac{p_1}{p_2} \times \frac{Q_1^d}{Q_2^d} + \tau^{-\phi}}{\tau^{-\phi}\left(\left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi}+1}{\left(\frac{p_1}{p_2}\right)^{-\phi}+\tau^{-\phi}}\right) \times \frac{p_1}{p_2} \times \frac{Q_1^d}{Q_2^d}\right) + 1} \tag{62}$$

Similarly, dividing Q_1^s by Q_2^s delivers the following relative supply equation:

$$\frac{Q_1^s}{Q_2^s} = \left(\frac{p_1}{p_2}\right)^{\psi} \times \left(\frac{\tau^{-\phi}\left(\frac{p_1}{p_2}\right)^{-\phi} + 1}{\left(\frac{p_1}{p_2}\right)^{-\phi} + \tau^{-\phi}} \right)^{-\frac{\psi}{\phi}} \tag{63}$$

Note that given the trade friction τ and gravity constants, the relative demand and relative supply can be solved solely as a function of relative output price $\frac{p_1}{p_2}$ using equations (62) and (63), allowing us to analytically characterize the equilibria using standard (relative) supply and demand curves.

Figure 3 depicts example equilibria possible for different combinations of gravity constants; the points where the two curves intersect are possible equilibria. The top left figure shows that when the supply and demand elasticities are both positive (corresponding to a case where the relative aggregate supply is increasing and the relative aggregate demand is decreasing), there is a unique equilibrium. The top right figure shows that when the supply elasticity is positive but the demand elasticity is negative, both the relative aggregate supply and demands are increasing, potentially resulting in multiple equilibria. Similarly, the bottom left figure shows that when the supply elasticity is negative and the demand elasticity is positive, both the relative aggregate supply and demand curves are decreasing, also potentially resulting in multiple equilibria. Finally, the bottom right figure shows that when both the supply and demand elasticities are negative and suitably large in magnitude, the relative aggregate supply curve is downward sloping and the relative aggregate demand curve is upward sloping, allowing for a unique equilibria (albeit one without much economic relevance). These examples are consistent with the sufficient conditions for uniqueness presented in Theorem 1.

B.8 Tariffs in the universal gravity framework

In this subsection, we show how one can use the tools developed above to analyze the effect of tariffs in a simple Armington trade model.

Because tariffs introduce an additional source of revenue, they are are not strictly contained within the universal gravity framework. However, it turns out that the equilibrium structure of an Armington trade model with tariffs is mathematically equivalent to the equilibrium structure of the universal gravity framework. As a result, we can apply Theorems 1 and 2 almost immediately to the case of tariffs in this model.

To see this, consider a simple Armington trade model with N locations.[42] Each location $i \in S$ is endowed with its own differentiated variety and L_i workers who supply their unit labor inelastically and consume varieties from all locations with CES preferences and an elasticity of substitution σ. Suppose that trade is subject to technological iceberg trade frictions $\tau_{ij} \geq 1$ and ad-valorem tariffs $\tilde{t}_{ij} \geq 0$. Define $t_{ij} \equiv 1 + \tilde{t}_{ij}$. Then we can write the value of trade flows from i to j (excluding the tariffs) as:

$$X_{ij} = \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma} A_i^{\sigma-1} w_i^{1-\sigma} P_j^{\sigma-1} E_j, \tag{64}$$

where A_i is the productivity in location $i \in S$, w_i is the wage, P_j is the ideal Dixit-Stiglitz price index, and E_j is expenditure.

Income in location i from trade is equal to its total sales (excluding tariffs):

$$Y_i = \sum_{j \in S} X_{ij}. \tag{65}$$

Total income (and hence expenditure) also includes the revenue earned from tariffs T_i:

$$E_i = Y_i + T_i, \tag{66}$$

[42] We consider an Armington model in order to have an explicit welfare function, the results that follow will hold for any general equilibrium model where the aggregate supply elasticity $\psi = 0$.

where tariff revenue is equal to the bilateral tariff charged on all trade being sent[43]:

$$T_i = \sum_{j \in S} \tilde{t}_{ji} X_{ji}. \tag{67}$$

The total expenditure by consumers in location i is also equal to its total imports plus the tariffs incurred:

$$E_i = \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji}. \tag{68}$$

Combining equations (66), (67), (68), we can demonstrate that trade flows are balanced:

$$\begin{aligned} E_i &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i + \sum_{j \in S} \tilde{t}_{ji} X_{ji} &= \sum_{j \in S} \left(1 + \tilde{t}_{ji}\right) X_{ji} \iff \\ Y_i &= \sum_{j \in S} X_{ji} \end{aligned} \tag{69}$$

Finally, total expenditure is equal to the payment to workers plus tariff revenue:

$$\begin{aligned} E_i &= w_i L_i + T_i \iff \\ Y_i &= w_i L_i \end{aligned} \tag{70}$$

Define $K_{ij} \equiv \tau_{ij}^{1-\sigma} t_{ij}^{-\sigma}$ as the bilateral "kernel", $B_i \equiv A_i L_i$ as the "income shifter", $\gamma_i \equiv A_i^{\sigma-1} w_i^{1-\sigma}$ as the origin fixed effect, $\delta_j \equiv P_j^{\sigma-1} E_j$ as the destination fixed effect, and $\alpha \equiv \frac{1}{1-\sigma}$. Combining equations (65), (69), and (70) yields the following system of equilibrium equations:

$$\begin{aligned} w_i L_i &= \sum_{j \in S} X_{ij} \iff \\ B_i \gamma_i^{\alpha} &= \sum_{j \in S} K_{ij} \gamma_i \delta_j \end{aligned} \tag{71}$$

$$\begin{aligned} w_i L_i &= \sum_{j \in S} X_{ji} \iff \\ B_i \gamma_i^{\alpha} &= \sum_{j \in S} K_{ji} \gamma_j \delta_i. \end{aligned} \tag{72}$$

Equations (71) and (72) can be jointly solved to recover the equilibrium $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$; given $\{\gamma_i\}_{i \in S}$ and $\{\delta_i\}_{i \in S}$, in turn, we can solve for all endogenous variables, as wages can be written as $w_i = \gamma_i^{\frac{1}{1-\sigma}} A_i$, the price index can be written as $P_i = \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{1}{1-\sigma}}$, expenditure can be written as $E_i = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)$, and real expenditure can be written as $W_i \equiv \frac{E_i}{P_i} = \delta_i \left(\sum_{j \in S} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} \gamma_j\right)^{\frac{\sigma}{\sigma-1}}$. As we note at the beginning of Section 3, this equilibrium system is

[43] If we had instead supposed that tariffs are only levied on goods that actually arrive, we would have $T_i = \sum_j \frac{\tilde{t}_{ji}}{\tau_{ji}} X_{ji}$, which does not change the following analysis in any substantive way.

identical in mathematical structure to the universal gravity equilibrium equations 6 and 7. Hence, Theorem 1 applies directly (with existence as long as $\sigma \neq 0$ and uniqueness as long as $\sigma \geq 1$). Moreover, a similar methodology as employed in Theorem 2 can be used to determine how the equilibrium variables γ_i and δ_i respond to shocks that alter the kernel K_{ij} (be they due to changes in iceberg trade frictions or tariffs). In particular:

$$\frac{\partial \ln \gamma_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^{+}_{l,i} + A^{+}_{N+l,j} - c\right) \tag{73}$$

$$\frac{\partial \ln \delta_l}{\partial \ln K_{ij}} = X_{ij} \times \left(A^{+}_{N+l,i} + A^{+}_{l,j} - c\right), \tag{74}$$

where $\tilde{A}^{-1}_{i,j}$ is the $\langle i,j\rangle$ element of the $2N \times 2N$ matrix the (pseudo) inverse $\tilde{\mathbf{A}}^{-1}$:[44]

$$\tilde{\mathbf{A}}^{-1} = \begin{pmatrix} \frac{\sigma}{1-\sigma}\mathbf{Y} & -\mathbf{X} \\ \frac{1}{1-\sigma}\mathbf{Y} - \mathbf{X}^{T} & -\mathbf{Y} \end{pmatrix}^{-1}, \tag{75}$$

Because all endogenous variables in the model are simple functions $\{\gamma_i\}_{i\in S}$ and $\{\delta_i\}_{i\in S}$, one can apply equations (73) and (74) to immediately derive any elasticity of interest, e.g. the effect of welfare in location l from changing the tariffs j impose on goods coming from i.

B.9 Global shocks

In this subsection we show that the "exact hat algebra" pioneered by Dekle et al. (2008) and extended by Costinot and Rodriguez-Clare (2013) can be applied to any model in the universal gravity framework to calculate the effect of any (possibly large) trade shock. (Note that Section 4 instead showed how to calculate the *elasticity* of endogenous variables to any trade friction shock). We show that the key takeaway from Section 4 holds for all trade shocks: Given observed data, all the gravity models with the same gravity constants imply the same counterfactual predictions for all endogenous variables (i.e. output prices, price indices, nominal incomes, real expenditures, and trade flows).

Consider an arbitrary change in the trade friction matrix $\{\tau_{ij}\}_{S\times S}$. In what follows, we denote with a hat the ratio of the counterfactual to initial value of the variable, i.e. $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}}$. The following proposition provides an analytical expression relating the change in the output price and the associated price index to the change in trade frictions and the initial observed trade flows:

Proposition 4. *Consider any given set of observed trade flows* $\mathbf{X}$*, gravity constants* ϕ *and* ψ*, and change in the trade friction matrix* $\widehat{\tau}$*. Then the percentage change in the exporter and importer shifters,* $\{\hat{p}_i\}$ *and* $\left\{\hat{P}_i\right\}$*, if it exists, will solve the following system of equations:*

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \sum_{j\in S}\frac{X_{ij}}{Y_i}\widehat{\tau}_{ij}^{-\phi}\widehat{P}_j^{\phi}\widehat{p}_j\left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi} \quad \textit{and} \quad \widehat{P}_i^{-\phi} = \sum_{j\in S}\left(\frac{X_{ji}}{E_j}\right)\hat{\tau}_{ji}^{-\phi}\hat{p}_j^{-\phi},\ \forall i\in S \tag{76}$$

Proof. We first note that equilibrium equations (10) and (7) must hold for both the initial and

[44] The psuedo-inverse can be calculated simply by removing the first row and column and taking the inverse; see footnote 18.

counterfactual equilibria. Taking ratios of the counterfactual to initial values yields:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \frac{\sum_{j\in S}\left(\tau'_{ij}\right)^{-\phi}\left(P'_j\right)^{\phi}p'_jC_j\left(\frac{p'_j}{P'_j}\right)^{\psi}}{\sum_{j\in S}\tau_{ij}^{-\phi}P_j^{\phi}p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi}}\forall i\in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S}\left(\tau'_{ji}\right)^{-\phi}\left(p'_j\right)^{-\phi}}{\sum_{j\in S}\tau_{ji}^{-\phi}p_j^{-\phi}},\ \forall i\in S$$

where we denote the counterfactual equilibrium variables with a prime and the initial equilibrium variables as unadorned. Note that from the gravity equation (10) (and C. 3 - C. 5) we have $X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi}$, where $p_jC_j\left(\frac{p_j}{P_j}\right)^{\psi} = E_j$, so that the above equations become:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \frac{\sum_{j\in S}\left(\tau'_{ij}\right)^{-\phi}\left(P'_j\right)^{\phi}p'_jC_j\left(\frac{p'_j}{P'_j}\right)^{\psi}}{p_i^{\phi}\sum_{j\in S}X_{ij}}\forall i\in S$$

$$\widehat{P}_i^{-\phi} = \frac{\sum_{j\in S}\left(\tau'_{ji}\right)^{-\phi}\left(p'_j\right)^{-\phi}}{P_i^{-\phi}\frac{1}{E_i}\sum_{j\in S}X_{ji}},\ \forall i\in S$$

Finally, note that from C. 2 and C. 4 we have $E_i = \sum_{j\in S}X_{ij}$ and $Y_i = \sum_{j\in S}X_{ij}$, respectively. Then using our definition $\hat{x}_i \equiv \frac{x_i^{counterfactual}}{x_i^{initial}} \iff x_i^{counterfactual} = \hat{x}_i x_i^{initial}$ we have:

$$\widehat{p}_i^{1+\phi+\psi}\widehat{P}_i^{-\psi} = \sum_{j\in S}\left(\frac{X_{ij}}{Y_i}\right)\hat{\tau}_{ij}^{-\phi}\hat{P}_j^{\phi}\hat{p}_j\left(\frac{\widehat{p}_j}{\widehat{P}_j}\right)^{\psi}\ \forall i\in S$$

$$\widehat{P}_i^{-\phi} = \sum_{j\in S}\left(\frac{X_{ji}}{E_j}\right)\hat{\tau}_{ji}^{-\phi}\hat{p}_j^{-\phi}\forall i\in S,$$

as required. □

Note that equation (76) inherits the same mathematical structure as equations (6) and (7). As a result, part (i) of Theorem 1 proves that there will exist a solution to equation (76) and part (ii) of Theorem 1 provides conditions for its uniqueness.

B.10 Identification

In this subsection, we show how one can always choose a set of bilateral trade frictions to match observed trade flows for any choice of gravity constants, own trade frictions, and supply shifters. We first state the result as a proposition before providing a proof.

Proposition 5. *Take as given the set of observed trade flows $\{X_{ij}\}$, an assumed set of supply shifters $\{C_i\}$, an aggregate scalar κ, and own trade frictions $\{\tau_{ii}\}$, and the gravity constants ϕ and ψ. Then there exists a unique set of trade frictions $\{\tau_{ij}\}_{i\neq j}$, output prices $\{p_i\}$, price indices $\{P_i\}$, and output $\{Q_i\}$ such that the following equilibrium conditions hold:*

1. *For all locations $i \in S$, income is equal to the product of the output price and theoutput:*

$$Y_i = p_i Q_i$$

2. *For all location pairs $i, j \in S$, the value of trade flows from i to j can be written in the following gravity equation form:*

$$X_{ij} = \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j$$

3. *For all locations $i \in S$, output satisfies the following supply condition:*

$$Q_i = \kappa C_i \left(\frac{p_i}{P_i} \right)^{\psi}$$

Proof. First, note that the income $Y_i = \sum_{j \in S} X_{ij}$, expenditure $E_i = \sum_{j \in S} X_{ji}$, and own expenditure share $\lambda_{jj} \equiv \frac{X_{jj}}{E_j}$, are all immediately derived from the observed trade flow data.

Second, let us define our unknown parameters and endogenous variables as functions of data and known parameters. The trade frictions are defined follows:

$$\tau_{ij} = \tau_{jj} \left(\frac{Y_j}{Y_i} \right) \left(\frac{\lambda_{jj}}{\lambda_{ii}} \right)^{\frac{\psi}{\phi}} \left(\frac{C_i}{C_j} \right) \left(\frac{\tau_{jj}}{\tau_{ii}} \right)^{\psi} \left(\frac{X_{jj}}{X_{ij}} \right)^{\frac{1}{\phi}}$$

for all $i, j \in S$ such that $i \neq j$.

The output prices are defined as

$$p_i = Y_i \left(\lambda_{ii} \tau_{ii}^{\phi} \right)^{\frac{\psi}{\phi}} / \kappa C_i$$

for all $i \in S$.

Given the output prices and trade frictions, the price index is defined as: for all $i \in S$,

$$P_i = \left(\sum_{j \in S} \tau_{ji}^{-\phi} p_j^{-\phi} \right)^{-\frac{1}{\phi}}.$$

Finally, the output in each location is defined as: for all $i \in S$,

$$Q_i = \kappa C_i \left(\frac{p_i}{P_i} \right)^{\psi}.$$

It is first helpful to note that given the above definitions of the trade frictions and output price indices, we have the following convenient relationship between own expenditure shares and prices:

$$\lambda_{jj} = \left(\tau_{jj} \frac{p_j}{P_j} \right)^{-\phi}$$

To see this, note that we can write:

$$
\begin{aligned}
\lambda_{jj} &= \left(\tau_{jj}\frac{p_j}{P_j}\right)^{-\phi} \iff \\
\frac{X_{jj}}{E_j} &= \frac{\tau_{jj}^{-\phi}p_j^{-\phi}}{\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi}} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\tau_{ij}^{-\phi}p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \left(\frac{X_{jj}}{E_j}\right)\sum_{i\in S}\left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{C_i}{C_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi}p_i^{-\phi} \iff \\
\tau_{jj}^{-\phi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)\left(\frac{(Y_i/C_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}}{(Y_j/C_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}}\right)\tau_{jj}^{-\phi}p_i^{-\phi} \iff \\
(Y_j/C_j)^{\phi}\left(\lambda_{jj}\tau_{jj}^{\phi}\right)^{\psi}p_j^{-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)(Y_i/C_i)^{\phi}\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\psi}p_i^{-\phi} \iff \\
p_j^{\phi-\phi} &= \sum_{i\in S}\left(\frac{X_{ij}}{E_j}\right)p_i^{\phi-\phi} \iff \\
E_j &= \sum_{i\in S}X_{ij},
\end{aligned}
$$

which is the definition of E_j.

We now confirm each of the three equilibrium conditions. To see that income is equal to the product of the output price and the output, we write:

$$
\begin{aligned}
p_i\times Q_i &= Y_i\times\left(\left(\lambda_{ii}\tau_{ii}^{\phi}\right)^{\frac{\psi}{\phi}}/\kappa C_i\right)\times Q_i \iff \\
p_i\times Q_i &= Y_i\times\left(\kappa C_i\left(\frac{p_i}{P_i}\right)^{\psi}\right)^{-1}\times Q_i \iff \\
p_i\times Q_i &= Y_i\times\frac{Q_i}{Q_i} \iff \\
p_i\times Q_i &= Y_i,
\end{aligned}
$$

as required.

To see that the value of trade flows can be written in the gravity equation form, we write the gravity equation as follows:

$$
\begin{aligned}
\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j &= \left(\tau_{jj}\left(\frac{Y_j}{Y_i}\right)\left(\frac{\lambda_{jj}}{\lambda_{ii}}\right)^{\frac{\psi}{\phi}}\left(\frac{C_i}{C_j}\right)\left(\frac{\tau_{jj}}{\tau_{ii}}\right)^{\psi}\left(\frac{X_{jj}}{X_{ij}}\right)^{\frac{1}{\phi}}\right)^{-\phi}p_i^{-\phi}P_j^{\phi}E_j \iff \\
\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j &= X_{ij}\left(\frac{(Y_i/C_i)^{\phi}\lambda_{ii}^{\psi}\tau_{ii}^{\phi\psi}}{(Y_j/C_j)^{\phi}\lambda_{jj}^{\psi}\tau_{jj}^{\phi\psi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}
\end{aligned}
$$

Recall from above that we have the following relationship between prices and own expenditure shares:

$$\lambda_{ii} = \left(\tau_{ii}\frac{p_i}{P_i}\right)^{-\phi}$$

so that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i)^{\phi}\left(\left(\frac{p_i}{P_i}\right)^{\psi}C_i\right)^{-\phi}}{(Y_j)^{\phi}\left(\left(\frac{p_j}{P_j}\right)^{\psi}C_j\right)^{-\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

Furthermore, recall that we have defined our quantities as follows:

$$Q_i = \kappa C_i\left(\frac{p_i}{P_i}\right)^{\psi},$$

which implies that:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\left(\frac{(Y_i/Q_i)^{\phi}}{(Y_j/Q_j)^{\phi}}\right)\left(\frac{p_i}{p_j}\right)^{-\phi}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We have shown above that $p_iQ_i = Y_i$, so that we have:

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

We claim that this implies that observed trade flows are explained by the gravity equation, i.e.:

$$X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$$

To see this, suppose not. Then we have

$$\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j = X_{ij}\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{X_{jj}}$$

but $X_{ij} \neq \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j$. Then without loss of generality we can write $X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j\varepsilon_{ij}$, where $\varepsilon_{ij} \neq 1$.

$$\begin{aligned}
\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j &= \left(\tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j\varepsilon_{ij}\right)\frac{\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j}{\left(\tau_{jj}^{-\phi}p_j^{-\phi}P_j^{\phi}E_j\varepsilon_{jj}\right)} \iff \\
1 &= \frac{\varepsilon_{ij}}{\varepsilon_{jj}} \iff \\
\varepsilon_{ij} &= \varepsilon_{jj} \equiv \varepsilon_j \ \forall i \in S
\end{aligned}$$

which then implies that we have:

$$X_{ij} = \tau_{ij}^{-\phi}p_i^{-\phi}P_j^{\phi}E_j\varepsilon_j$$

however, we know that:

$$
\begin{aligned}
\sum_{i \in S} X_{ij} = E_j &\iff \\
\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi} P_j^{\phi} E_j \varepsilon_j = E_j &\iff \\
\frac{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}}{\sum_{i \in S} \tau_{ij}^{-\phi} p_i^{-\phi}} = \frac{1}{\varepsilon_j} &\iff \\
\varepsilon_j = 1, &
\end{aligned}
$$

which is a contradiction. Hence, the observed trade flows are explained by the gravity equation.

Finally, we note that the third equilibrium condition trivially holds by the definition of Q_i:

$$
Q_i = \kappa C_i \left(\frac{p_i}{P_i} \right)^{\psi}.
$$

Hence, given our definitions, we have found a unique set of trade frictions $\{\tau_{ij}\}_{j \neq i}$, output prices $\{p_i\}_{i \in S}$, price indices $\{P_i\}_{i \in S}$, and output $\{Q_i\}_{i \in S}$ such that the equilibrium conditions hold for any set of observed trade flows $\{X_{ij}\}_{i,j \in S}$, an assumed set of supply shifters $\{C_i\}_{i \in S}$ and own trade frictions $\{\tau_{ii}\}_{i \in S}$, and the gravity constants (ϕ, ψ). □

B.11 Real output prices, welfare, and the openness to trade

In this section, we explore the relationship between the real output E_i/P_i and real output price p_i/P_i in the universal gravity framework and the welfare in a number of seminal models. We then show how the real output price in the universal gravity framework relates to the observed own expenditure share. Combining the two results allow ones to write the welfare in each of these models as a function of observed own expenditure share, as in Arkolakis et al. (2012a).

B.11.1 Real output prices and welfare

In this subsection, we provide a mapping between real output prices and the welfare of a unit of labor for the trade introduced and the economic geography model in Section 2.

The trade model In the trade model, the output price p_i is $w_i^{\zeta} P_i^{1-\zeta} / A_i$. As a result we have the welfare of each worker Ω_i can be expressed as a function of the real output price in the universal gravity framework as follows:

$$
\frac{w_i}{P_i} = \underbrace{\left(\frac{p_i A_i}{P_i^{1-\gamma}} \right)^{\frac{1}{\zeta}}}_{=w_i} \frac{1}{P_i} = A_i^{\frac{1}{\gamma}} \left(\frac{p_i}{P_i} \right)^{\frac{1}{\zeta}}.
$$

Or equivalently, we can express the welfare in terms of the supply elasticity.

$$
\frac{w_i}{P_i} = A_i^{1+\psi} \left(\frac{p_i}{P_i} \right)^{1+\psi}.
$$

The economic geography model In the economic geography model, the welfare is $\frac{w_i}{P_i}u_i$,and the price p_i is $\frac{w_i}{\overline{A}_i L_i^a}$. Therefore the welfare is

$$\Omega = \overline{A}_i \overline{u}_i L_i^{a+b} \left(\frac{p_i}{P_i}\right).$$

Welfare equalization and the labor market clearing condition implies

$$\Omega = \left(\overline{L}\right)^{a+b} \left[\sum_{i \in S} \left[\overline{A}_i \overline{u}_i \left(\frac{p_i}{P_i}\right)\right]^{-\frac{1}{a+b}}\right]^{-(a+b)}.$$

B.11.2 Real expenditure, real output prices and the openness to trade

In this subsection, we show we can express real expenditure and real output prices in any model within the universal gravity framework as a function of openness to trade and the gravity constants, as in Arkolakis et al. (2012a).

We begin by defining $\lambda_{ii} \equiv \frac{X_{ii}}{E_i}$ as location i's own expenditure share. From equation (10), we can express the real output price $\frac{p_i}{P_i}$ in a location as a function of its own expenditure share:

$$\begin{aligned} X_{ij} &= \frac{p_{ij}^{-\phi}}{\sum_{k \in S} p_{kj}^{-\phi}} E_j \implies \\ \frac{p_i}{P_i} &= \lambda_{ii}^{-\frac{1}{\phi}}. \end{aligned} \tag{77}$$

Moreover, given C. 3, C. 4 and C. 5, we can write total real expenditure $W_i \equiv \frac{E_i}{P_i}$ as a function of its own expenditure share as well:

$$\begin{aligned} W_i &= \frac{E_i}{P_i} \iff \\ W_i &= \left(\frac{p_i}{P_i}\right) Q_i \iff \\ W_i &= \left(\frac{p_i}{P_i}\right) \left(\kappa C_i \left(\frac{p_i}{P_i}\right)^{\psi}\right) \iff \\ W_i &= \kappa C_i \left(\frac{p_i}{P_i}\right)^{1+\psi}. \end{aligned} \tag{78}$$

Combining equations (77) and (78) yields:

$$W_i = \kappa C_i \left(\lambda_{ii}\right)^{-\frac{1+\psi}{\phi}}.$$

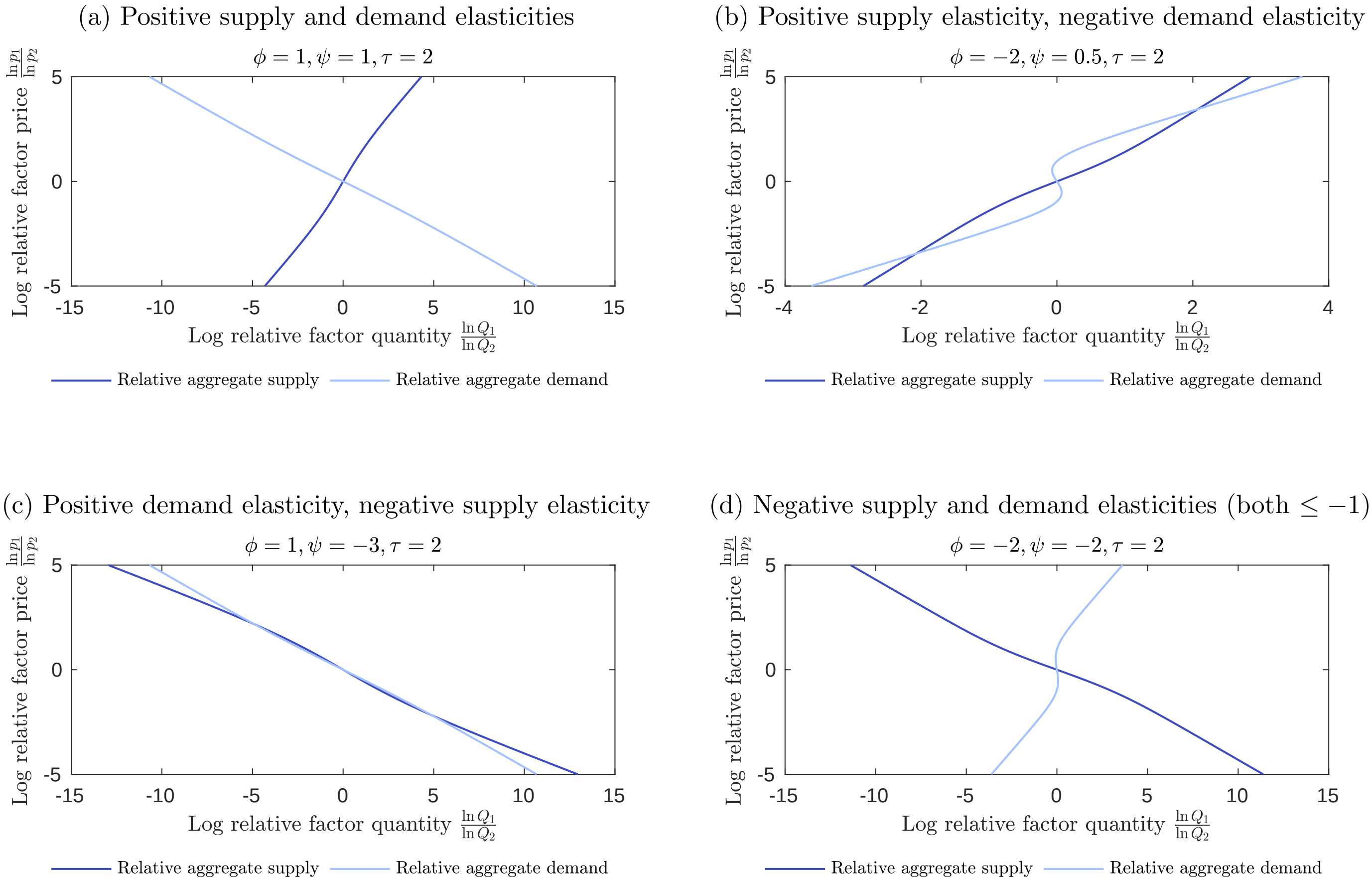
Note that a positive aggregate supply elasticity ($\psi > 0$) increases the elasticity of total real expenditure to own expenditure share, thereby amplifying the gains from trade. Note too that the derivations above imply that:

$$\frac{\partial \ln W_i}{\partial \ln \tau_{ij}} = (\psi + 1) \frac{\partial \ln \left(\frac{p_i}{P_i} \right)}{\partial \ln \tau_{ij}} + \frac{\partial \ln \kappa}{\partial \ln \tau_{ij}},$$

i.e. we can recover the elasticity of the total real expenditure (to-scale) to the trade friction shock from the elasticity of the real output price to the trade friction shock by simply multiplying by $\psi + 1$.

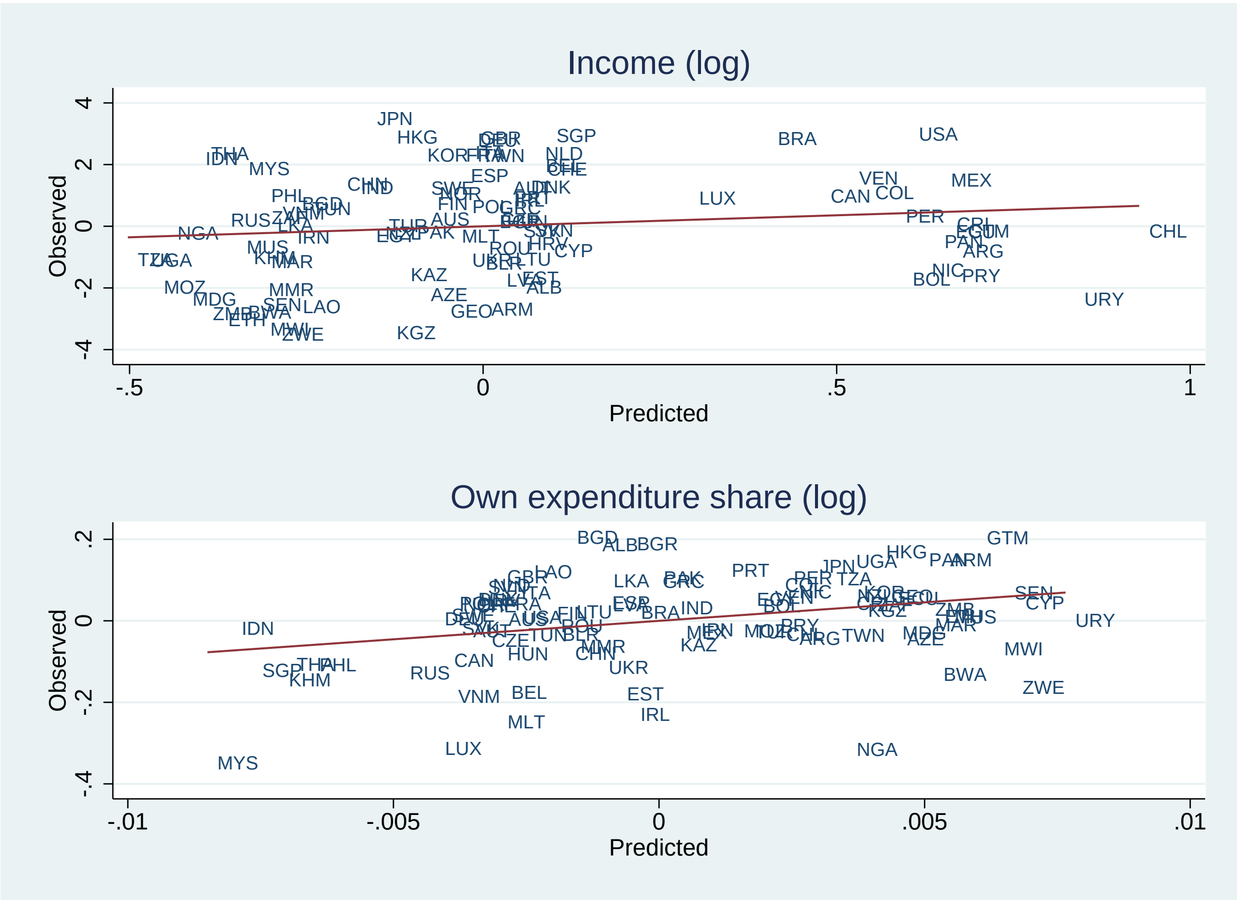
B.12 Additional Figures

Figure 3: Examples of multiplicity and uniqueness in two locations



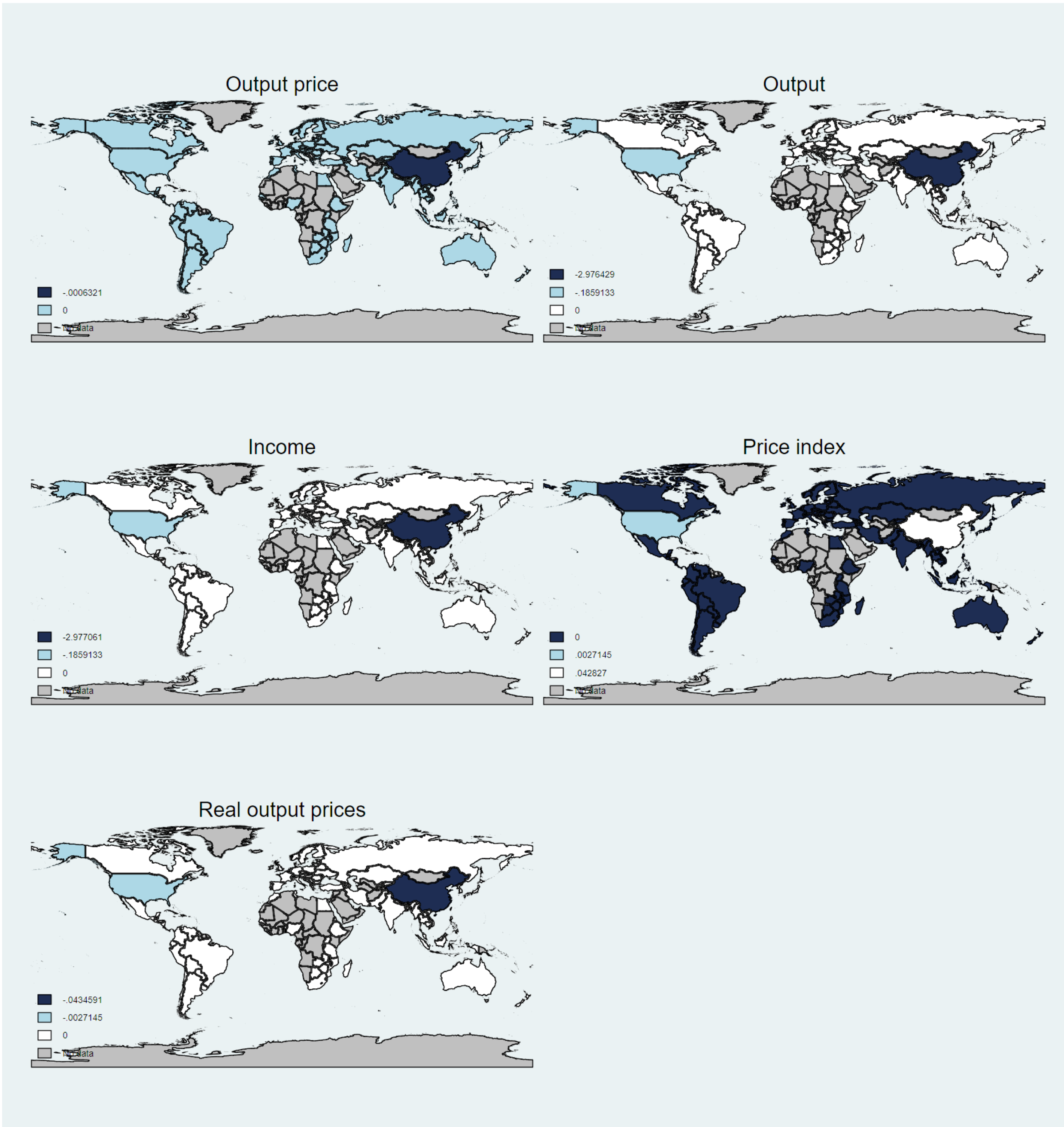
Notes: This figure shows examples of relative supply curve and relative demand curves for a two location world for different combinations of supply and demand elasticities; see Section B.7 for a discussion.

Figure 4: Correlation between observed income and own expenditure shares and the equilibrium values from the gravity model



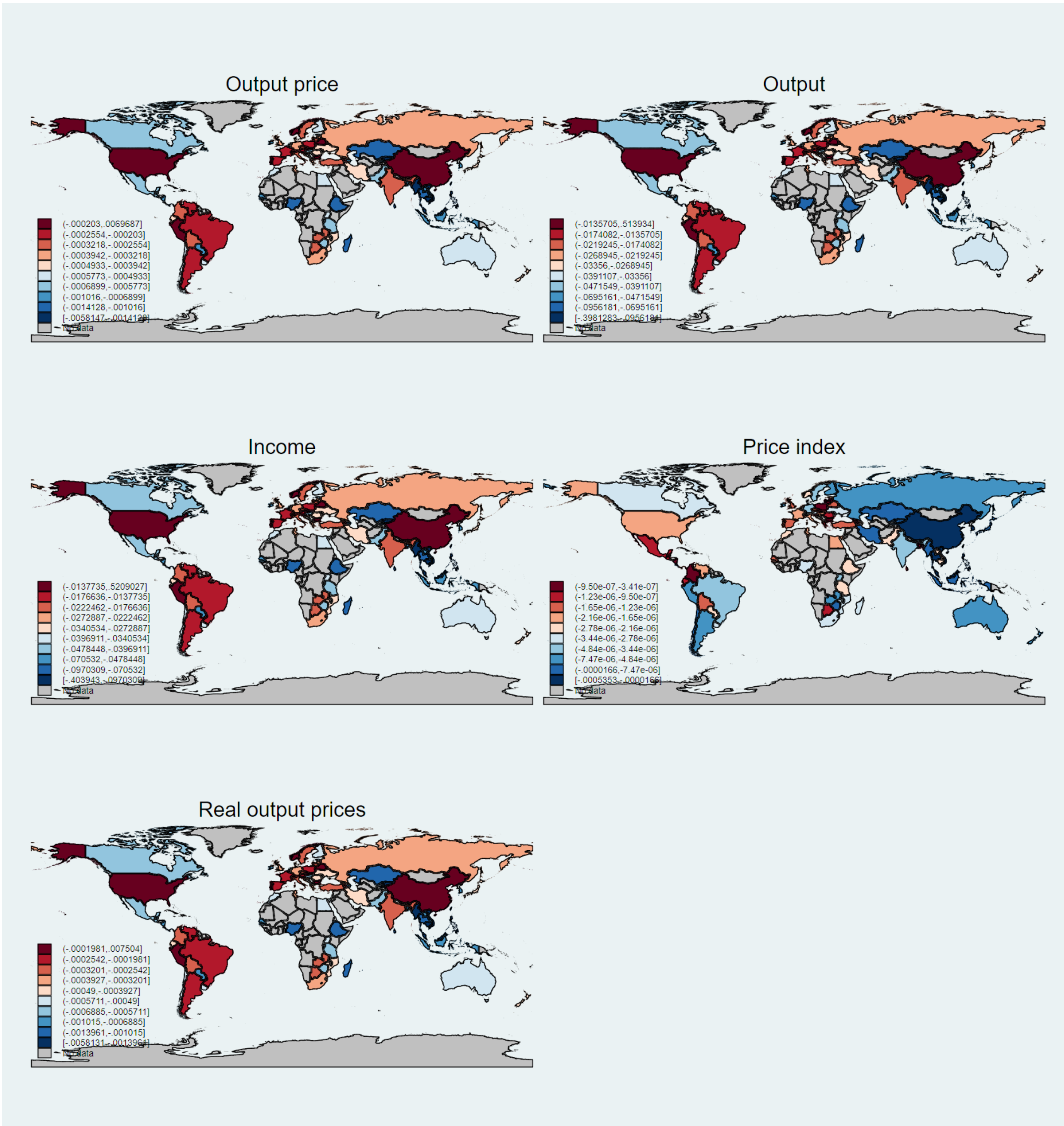
Notes: This figure shows the relationship between the observed and predicted income and own expenditure shares, respectively. The predicted incomes and own expenditure shares are the equilibrium values from the general equilibrium gravity model where bilateral frictions are those estimated from a fixed effects gravity regression and the supply shifters are estimated from a regression of log income on geographic and institutional controls. The scatter plots are plots of the residuals after controlling for the direct effect of the geographic, historical, and institutional observables.

Figure 5: The network effect of a U.S.-China trade war: Degree 0



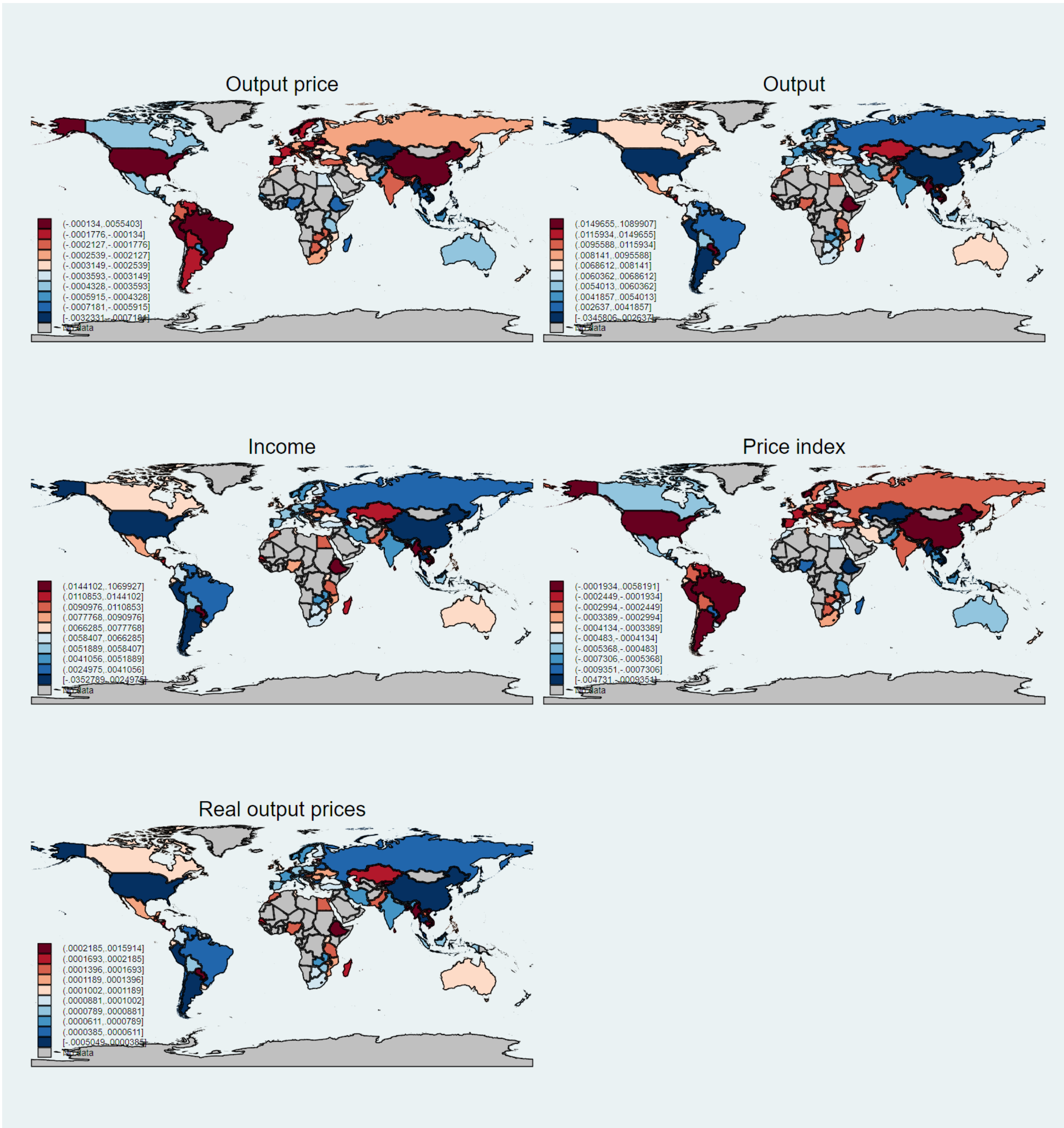
Notes: This figure depicts the "degree 0" effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The "degree 0" effect is the direct impact of the trade war on the U.S. and China, holding constant the price and output in all other countries. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 6: The network effect of a U.S.-China trade war: Degree 1



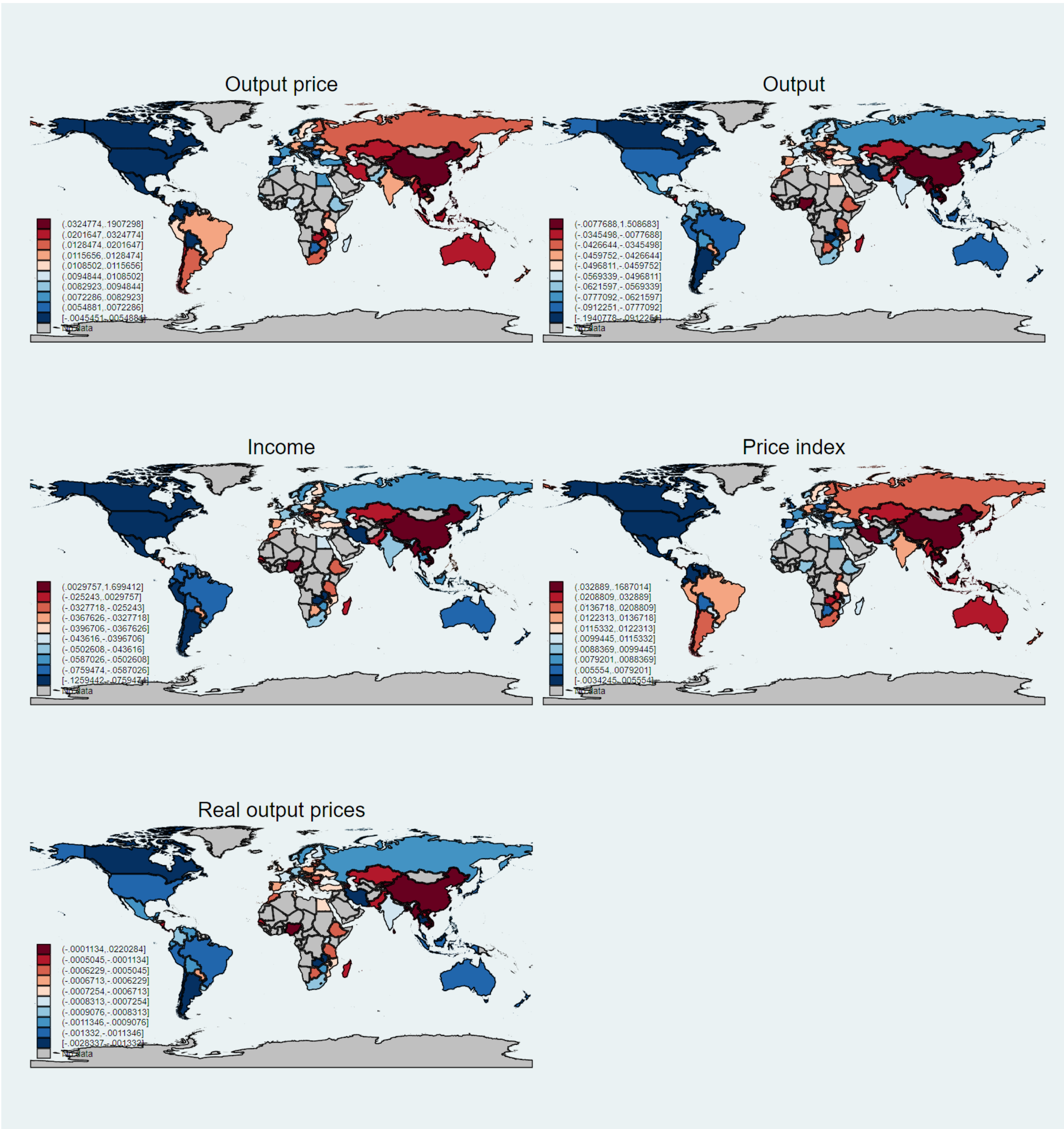
Notes: This figure depicts the "degree 1" effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The "degree 1" effect is the impact of the "degree 0" shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 7: The network effect of a U.S.-China trade war: Degree 2



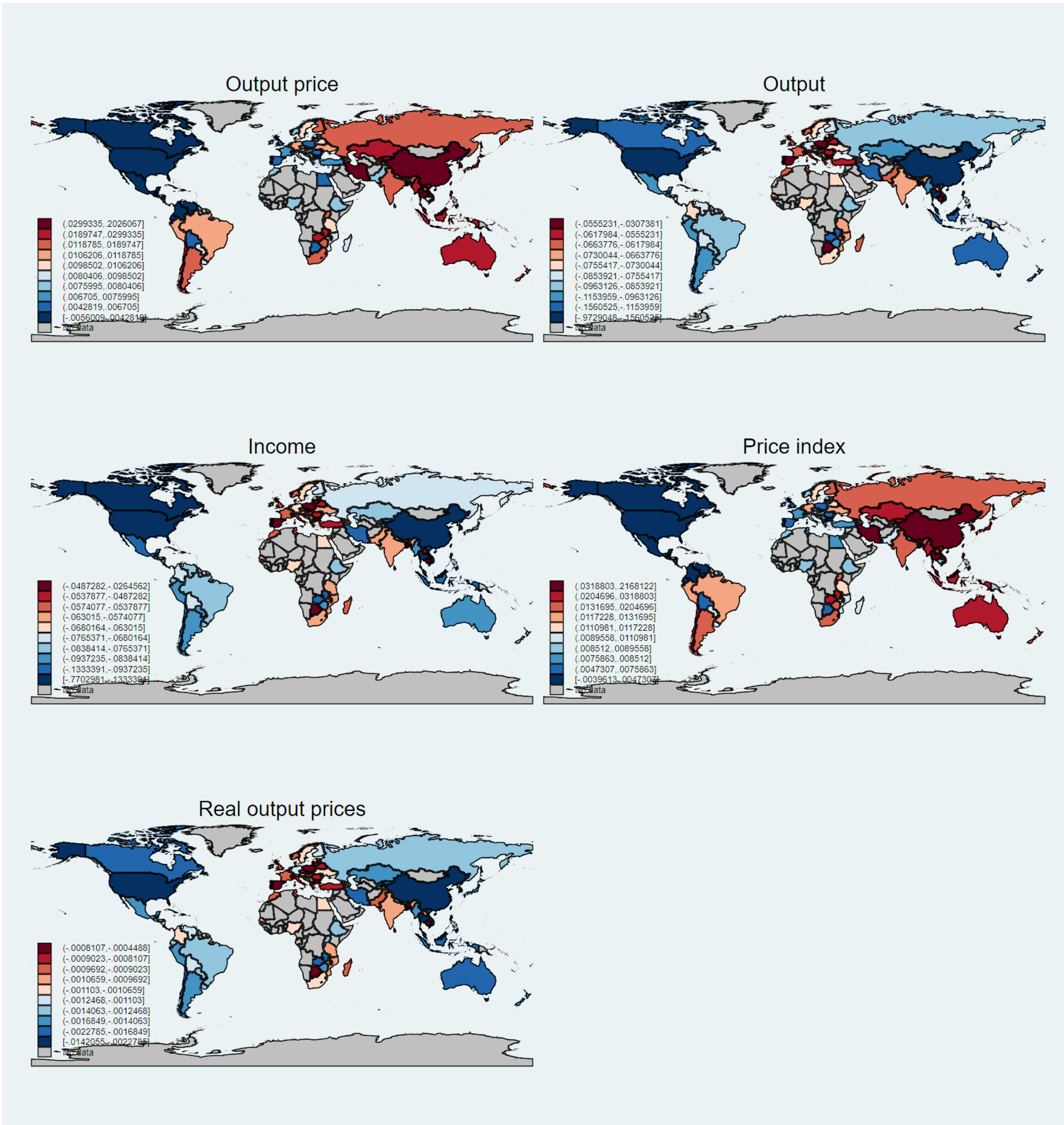
Notes: This figure depicts the "degree 2" effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. The "degree 2" effect is the impact of the "degree 1" shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 8: The network effect of a U.S.-China trade war: Degrees >2



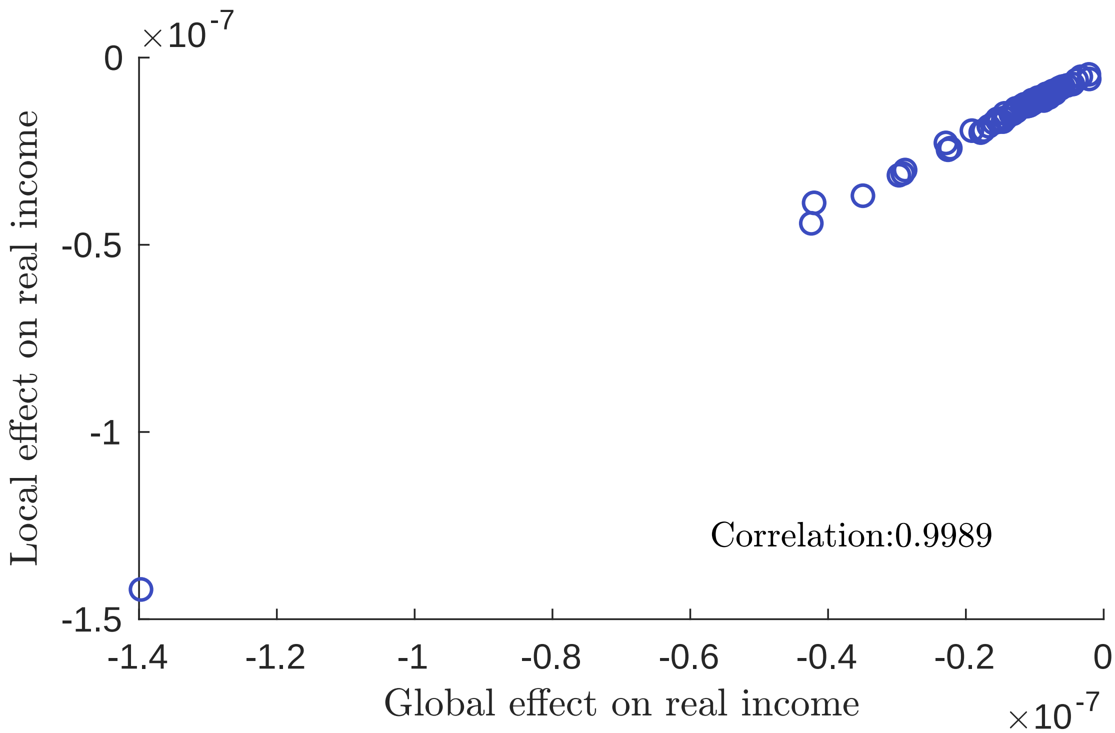
Notes: This figure depicts the cumulative effect of all degrees greater than two of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. A degree k effect is the impact of a degree $k-1$ shock on all countries through the trade network, holding constant the prices and output of their trading partners. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 9: The network effect of a U.S.-China trade war: Total effect



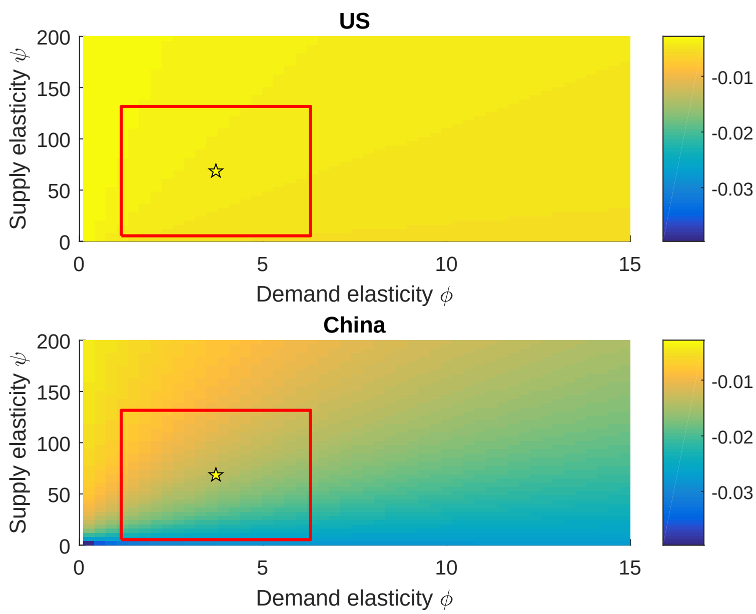
Notes: This figure depicts the total effect of an increase in the bilateral trade frictions between the U.S. and China (a "trade war") in all countries. This is the infinite sum of all degree k effects. Note that output prices, output, and the price index effects are identified only to scale, whereas the level of income and real output prices are known (see the discussion in Section 2).

Figure 10: Local versus global effects of a U.S.-China trade war



Notes: This figure depicts the correlation of the local (infinitesimal) elasticities and the global (50% increase) impacts of a trade war on the real output price in each country.

Figure 11: The effect of a U.S.-China trade war on real output prices in the U.S. and China: Robustness



Notes: This figure depicts the elasticity of real output prices to an increase bilateral trade frictions between the U.S. and China (a "trade war") for many constellations of demand and supply elasticities ϕ and ψ, respectively. The star indicates the estimated supply and demand elasticity constellation, and the red box outlines the 95% confidence interval of the two parameters.

Figure 12: Excess non-monotonic demand function for 1, $Z_1\left(p_2\right)$